Enrollment No: ______________________ Exam Seat No:______________________

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2 **Branch: B. Tech (All)**

Semester : 2 **Date : 12/09/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The interval of convergence of the logarithmic series $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - .....\infty$ is

(A) $-1 < x \le 1$ (B) $-1 < x < 2$ (C) $-\infty < x < \infty$ (D) $-1 \le x \le 1$

b) The series $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - .....\infty$ is

(A) convergent (B) divergent (C) finitely oscillating
(D) infinitely oscillating

c) The value of $\int_{-1}^{1} \sin^{11} x \, dx$

(A) 10! (B) $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{5} \cdot \frac{\pi}{2}$ (C) 0 (D) none of these

d) Let $f(b)$ be an odd function in the interval $\left[-\frac{T}{2}, \frac{T}{2}\right]$ with a period *T*,

then $F(x) = \int_{a}^{x} f(t) \, dt$ is

(A) periodic (B) non-periodic (C) periodic with period 2*T*
(D) periodic with period 4*T*

e) $\lceil 4.5 =$ ______

(A) $\frac{\sqrt{\pi}}{16}$ (B) $\frac{105\sqrt{\pi}}{16}$ (C) $\frac{5\sqrt{\pi}}{16}$ (D) none of these

f) $B(1, 1) =$ ______

(A) 1 (B) 0 (C) 1/2 (D) none of these

g) $\int_{-a}^{a} e^{-t^2} dt$ is equal to

(A) $\sqrt{\pi}\ erf(a)$ (B) $\sqrt{\pi}\ erf_c(a)$ (C) $\frac{\sqrt{\pi}}{2}\ erf(a)$ (D) $\frac{\sqrt{\pi}}{2}\ erf_c(a)$

h) $\int_0^{\frac{\pi}{2}} \sqrt{1-\frac{1}{4}\sin^2\theta}\ d\theta$ is equal to

(A) $E\left(\frac{1}{2}\right)$ (B) $E\left(\frac{1}{4}\right)$ (C) $K\left(\frac{1}{2}\right)$ (D) $K\left(\frac{1}{4}\right)$

i) If the two tangents at the point are real and distinct the double point is called

(A) a node (B) a cusp (C) a conjugate point (D) none of these

j) The curve $y^2(a+x)=x^2(a-x)$ where $a>0$ represent

(A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid
(D) Folium of Descartes

k) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx\ dy$ is equal to

(A) πa^2 (B) $\frac{\pi a^2}{2}$ (C) $\frac{\pi a^2}{4}$ (D) none of these

l) The transformations $x+y=u, y=uv$ transform the area element $dy\ dx$ into $|J|du\ dv$, where $|J|$ is equal to

(A) 1 (B) u (C) – 1 (D) none of these

m) The degree of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x\log\left(\frac{d^2y}{dx^2}\right)$ is

(A) 1 (B) 2 (C) 3 (D) none of these

n) The homogeneous differential equation $f_1(x,y)dx+f_2(x,y)dy=0$ can be reduced to a differential equation in which the variables are separated, by the substitution

(A) $y=vx$ (B) $x+y=v$ (C) $xy=v$ (D) $x-y=v$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Using reduction formula evaluate: $\int_0^{\pi} x\sin^7 x\cos^4 x\ dx$ **(5)**

b) Prove that $\int_0^2 x^4\left(8-x^3\right)^{-\frac{1}{3}} dx=\frac{16}{3}B\left(\frac{5}{3},\ \frac{2}{3}\right)$. **(5)**

c) Evaluate: $\int_0^{\frac{\pi}{2}}\int_0^{a\sin\theta}\int_0^{\frac{a^2-r^2}{a}} r\ dr\ d\theta\ dz$ **(4)**

Q-3 **Attempt all questions** **(14)**

	a)	Prove that $\int_0^{\frac{\pi}{2}} \frac{dx}{\tan^p x} = \frac{\pi}{2}\sec\left(\frac{p\pi}{2}\right)$.	(5)
	b)	Solve: $\left(xy^2 + e^{-\frac{1}{x^3}}\right)dx - x^2 y\, dy = 0$	(5)
	c)	Discuss the convergence of $\sum \frac{\sqrt{n+1}-\sqrt{n}}{n}$.	(4)
Q-4		**Attempt all questions**	**(14)**
	a)	By changing the transformations $x+y=u,\ y=uv$, show that $\int_0^1\int_0^{1-x} e^{\frac{y}{(x+y)}}\, dy\, dx = \frac{e-1}{2}$.	(5)
	b)	Show that $\sum_{n=1}^{\infty}\frac{1}{n^p} = \frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+.....+\frac{1}{n^p}+.....$ is (i) convergent if $p>1$ and (ii) divergent if $p \le 1$.	(5)
	c)	Using reduction formula evaluate: $\int_0^1 \frac{x^6}{\left(1+x^2\right)}\, dx$	(4)
Q-5		**Attempt all questions**	**(14)**
	a)	Solve: $\left(xy\sin xy + \cos xy\right)ydx + \left(xy\sin xy - \cos xy\right)xdy = 0$	(5)
	b)	By changing into polar co-ordinates, evaluate the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} \left(x^2+y^2\right)\, dx\, dy$	(5)
	c)	Evaluate: $\int_0^{\infty} e^{-h^2x^2} dx$	(4)
Q-6		**Attempt all questions**	**(14)**
	a)	Examine the series $\sum_{n=1}^{\infty}\frac{n!}{3^n}$ for convergence using ratio test.	(5)
	b)	Using reduction formula prove that $\int_0^a x^5\left(2a^2-x^2\right)^{-3} dx = \frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$.	(5)
	c)	Solve: $\left(x^2+y^2-a^2\right)x\, dx + \left(x^2-y^2-b^2\right)y\, dy = 0$	(4)
Q-7		**Attempt all questions**	**(14)**
	a)	Trace the curve $xy^2 = 4a^2\left(2a-x\right)$.	(5)
	b)	Show that the volume of the spindle-shaped solid generated by revolving the asteroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ about the x-axis is $\frac{32\pi a^3}{105}$.	(5)
	c)	Evaluate: $\int_1^{\infty}\frac{dx}{\sqrt{x^4-1}}$	(4)
Q-8		**Attempt all questions**	**(14)**
	a)	Prove that $erf_c\left(x\right)+erf_c\left(-x\right)=2$.	(5)
	b)	Trace the curve $r = a\left(1+\cos\theta\right)$.	(5)

c) Find the length of the arc of the Catenary $y = c \cosh\left(\frac{x}{c}\right)$ measured from the vertex $(0,\ c)$ to any point on the Catenary. **(4)**